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Thermophoretic Effect on Convective Heat and Mass Transfer Flow over a Vertical Porous Plate in a Rotating System with Suction/Injection

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Abstract

A mathematical model has been developed to investigate double diffusive convective heat and mass transfer flow over a vertical porous plate in a rotating system with thermophoretic effect. The governing equations pertaining to the model for mass, momentum, energy and species concentration are transformed into non-dimensional form by using similarity transformation. Numerical solutions are found by using shooting method that uses fourth order Runge-Kutta method and Newton-Raphson method. The field variables for primary and secondary velocity, energy and concentration are discussed for various values of rotational parameter, thermophoretic parameter and suction/injection parameter through graphically as well as skin-friction coefficient in x and z directions and local rate of heat and mass transfer coefficients are analyzed for various physical parameters and whose results are reported in tabular form.

Keywords: Thermophoretic effect; Vertical porous plate; Rotating Effect; Suction/Injection.

Introduction

Double diffusive convective flow of heat and mass transfer over a vertical flat plate in a rotating system is considerable interest in many industrial, technological and engineering applications such as in the design of turbo machines and turbines. The influence of mass transfer and free convection currents on the flow over a flat porous plate in a rotating system is investigated by Raptis and Perdiki [1]. Recently, Salah et.al [2] investigated unsteady MHD effects on convective flow of a second grade fluid in a porous medium in a rotating system. Chamkha and Ahmed [3] studied the effects of unsteady mixed convective heat and mass transfer flow in the forward stagnation region of a rotating sphere at different wall conditions. Bhuvanavijaya [4] analyzed the effects of higher order chemical reaction and heat sources on convective heat and mass transfer flow over a vertical plate in a rotating system. Bhuvanavijaya and Mallikarjuna [5] investigated convective heat and mass transfer flow past a vertical plate in a rotating system with temperature dependent thermal conductivity and variable porosity regime.

The studies in aerosol particle deposition caused by thermophoresis have gained immense important for many engineering and industrial applications for the last few decades. It shows that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition processed used in the fabrication of optical fiber. Alam et.al [6]

studied the influence of thermophoresis on MHD mixed convective heat and mass transfer flow past an inclined porous plate with heat source, viscous dissipation, Joule heating and chemical reaction. Grosen et.al [7] studied thermophoretic deposition of particles on mixed convective flow in a fully developed parallel-plate vertical channel. Hayat and Qasim [8] investigated the effect of thermophoresis and thermal radiation on MHD flow of a Maxwell fluid in the presence of Joule heating. Kandasamy et.al [9] used group theory transformation to analyze the effect of thermophoresis and cross diffusion on natural convective heat and mass transfer with chemical reaction and heat source/sink over a porous stretching surface. Noor et.al [10] studied thermophoretic effect on on MHD flow on convective heat and mass transfer over an inclined radiate permeable surface with heat source/sink. Ganesan et.al [11] investigated the effect of thermophoresis particle deposition in a natural convective doubly stratified medium over a vertical plate. Bhuvanavijaya and Mallikarjuna [12] studied thermophoresis effects on free convective heat and mass transfer flow over a vertical wavy surface in a fluid saturated porous medium in the presence of variable properties.

The authors are envisage to investigate the effect of thermophoresis on convective heat and mass flow over a vertical plate in a rotating system with

suction/injection. The governing equations for the conservation of mass, momentum, energy and species concentration are transformed into non-dimensional form by using appropriate similarity transformations. The transformed equations are solved by using numerical method namely, shooting method that uses Runge-Kutta fourth method and Newton-Raphson method. The results are reported graphically for various values of physical parameter, thermophoresis parameter, rotational parameter and suction parameter on fluid flow characteristics primary velocity, secondary velocity, temperature and concentration distributions.

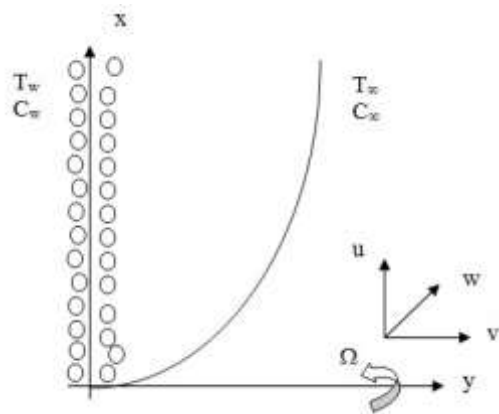


Fig 1: Physical Configuration and coordinate system

Formulation of the problem

We consider two dimensional, steady, viscous incompressible fluids over a vertical flat surface in a rotating system embedded in a fluid saturated porous medium. The physical configuration and coordination of the system is shown in figure-1. We assumed here that all physical quantities are depends on x and y . The vertical surface is placed along the x -direction and moving with a uniform velocity U_0 , and y -axis is taken normal to vertical surface. The uniform temperature T_w and concentration C_w are assumed along the plate, which are higher than free stream fluid temperature T_∞ and concentration C_∞ . The appropriate conservation equations with aforesaid assumption under the Boussinesq and boundary layer approximations can be shown to take the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g' \beta (T - T_\infty) + g' \beta^* (C - C_\infty) + 2\Omega w \quad (2)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + 2\Omega (U_0 - u) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} (V_T C) \quad (5)$$

The boundary conditions for this problem are;

$$\left. \begin{aligned} u=0, v=v_0(x), w=0, T=T_w, C=C_w \text{ at } y=0 \\ u=U_0, w=0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (6)$$

By using the work of Sattar [13], a transformation is assumed as,

$$u_1 = U_0 - u \Rightarrow u = U_0 - u_1$$

Eqs. (1) - (5) with boundary conditions (6), are transformed respectively into

$$-\frac{\partial u_1}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$\begin{aligned} (U_0 - u_1) \frac{\partial u_1}{\partial x} + v \frac{\partial u_1}{\partial y} = \nu \frac{\partial^2 u_1}{\partial y^2} + g' \beta (T - T_\infty) \\ + g' \beta^* (C - C_\infty) + 2\Omega w \end{aligned} \quad (8)$$

$$(U_0 - u_1) \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + 2\Omega u_1 \quad (9)$$

$$(U_0 - u_1) \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (10)$$

$$(U_0 - u_1) \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} (V_T C) \quad (11)$$

with subject to the boundary conditions are

$$\left. \begin{aligned} u_1 = U_0, v = v_0(x), w = 0, T = T_w, C = C_w \text{ at } y = 0 \\ u_1 = 0, w = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (12)$$

where u, v, w are the velocity components in the direction x, y, z respectively, β is the coefficient of volumetric thermal expansion, β^* is the volumetric mass expansion, g' is the acceleration due to gravity, ρ is the density, ν is the kinematics viscosity, k is the thermal conductivity of the medium, T, T_w, T_∞ , are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively. C, C_w, C_∞ are the concentration of the fluid inside the concentration boundary layer, the plate concentration and the fluid concentration in the free stream respectively and other symbols have their usual meaning.

In Eq. [11] V_T is the thermophoretic velocity which can be written as (see Wu and Greif [14])

$$V_T = -\frac{kv}{T_r} \frac{\partial T}{\partial y} \quad (13)$$

where k is thermophoretic coefficient which ranges in the values between 0.2 and 1.2 and is defined as (see Talbot et.al [15])

$$k = \frac{2C_s(\lambda_g / \lambda_p + C_t Kn) [1 + Kn(C_1 + C_2 e^{-C_3 / Kn})]}{(1 + 3C_m Kn)(1 + 2\lambda_g / \lambda_p + 2C_t Kn)}$$

where $C_1, C_2, C_3, C_m, C_s, C_t$ are constants, λ_g and λ_p are thermal conductivities of fluid and diffused particles, respectively and Kn is the Knudsen number.

To render dimensionless solutions and facilitate numerical analysis, we define the following dimensionless variables:

$$\eta = y \sqrt{\frac{U_0}{2\nu x}}, \quad g(\eta) = \frac{w}{U_0}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{\bar{x}(C_0 - C_\infty)}, \quad \psi = \sqrt{2\nu x U_0} f(\eta)$$

$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{\nu U_0}{2x}} [\eta f'(\eta) - f(\eta)],$$

$$u_1 = \frac{\partial \psi}{\partial y} = U_0 f'(\eta) \Rightarrow \frac{u}{U_0} = 1 - f'(\eta) \quad (14)$$

Now the plate of concentration is assumed as

$$C_w(x) = C_\infty + \bar{x}(C_0 - C_\infty), \quad \text{where } C_0 \text{ is the mean concentration and } \bar{x} = \frac{x U_0}{\nu} \quad (15)$$

where f_w is the suction parameter or transpiration parameter.

Making use of equations (13) – (15) the governing equations (7) – (11) with boundary conditions (12) are transformed into

$$f''' + (\eta - f)f'' - g_s \theta - g_c \phi - Rg = 0 \quad (16)$$

$$g'' + (\eta - f)g' + Rg' = 0 \quad (17)$$

$$\theta'' + \text{Pr}(\eta - f)\theta' = 0 \quad (18)$$

$$\phi'' + \text{Sc}(\eta - f - \tau\theta')\phi' - \text{Sc}\tau\phi\theta'' = 0 \quad (19)$$

$$\left. \begin{aligned} f = f_w, f' = 1, g = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \\ f' = 0, g = 0, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (20)$$

where

$$Gr = \frac{g' \beta (T_w - T_\infty) 2x^3}{\nu^2}, \quad Gm = \frac{g' \beta^* (C_w - C_\infty) 2x^3}{\nu^2},$$

$$Sc = \frac{\nu}{D_m}, \quad \tau = -\frac{k}{T_r} (T_w - T_\infty), \quad R = \frac{4\Omega x}{U_\infty}$$

$$, \quad g_s = \frac{Gr}{\text{Re}^2}, \quad g_c = \frac{Gm}{\text{Re}^2} \quad (21)$$

Here Gr is the Grashof number, Gm is the modified Grashof number and R is the rotational parameter, Pr is the Prandtl number, Sc is the Schmidt number is the temperature buoyancy parameter, g_s is the thermal buoyancy parameter, g_c is the mass buoyancy parameter and τ is the thermophoresis parameter.

The physical quantities of interest are the skin friction co-efficient C_f , which is defined as

$$C_f = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad \mu \left(\frac{\partial w}{\partial y} \right)_{y=0} \quad \text{which are}$$

transformed into non-dimensional form:

$$\tau_x = \left(\frac{\partial^2 f}{\partial \eta^2} \right)_{\eta=0} \quad \text{and} \quad \tau_z = \left(\frac{\partial g}{\partial \eta} \right)_{\eta=0} \quad (22)$$

The Nusselt number is denoted by

$$Nu = -\frac{1}{\Delta T} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad \text{is transformed into non-}$$

$$\text{dimensional form: } \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} \quad (23)$$

The Sherwood number is denoted by

$$Sh = -\frac{1}{\Delta C} \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad \text{is transformed into non-}$$

$$\text{dimensional form: } \left(\frac{\partial \phi}{\partial \eta} \right)_{\eta=0} \quad (24)$$

Numerical method

The equations (16)-(19) are converted into the following system of linear differential equations of first order

$$f' = L, \quad f'' = M, \quad g' = N,$$

$$\theta' = P, \quad \phi' = Q$$

$$f''' = (f - \eta)M + g_s \theta + g_c \phi + Rg$$

$$g'' = (f - \eta)N - RL$$

$$\theta'' = (f - \eta)Pr P$$

$$\phi'' = \text{Sc}(f - \eta + \tau P)Q + \text{Sc}\tau\phi((f - \eta)Pr P) \quad (25)$$

The boundary conditions are

$$f(0) = f_w, \quad L(0) = 1, \quad g(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (26)$$

$$L(\infty) = 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0$$

As the initial values for $M(0) = f''(0)$, $N(0) = g'(0)$, $P(0) = \theta'(0)$ and

$Q(0) = \phi'(0)$ are not specified in the boundary conditions, assume some suitable values for $M(0)$,

$N(0)$, $P(0)$ and $Q(0)$. Then the eqs. (25) are integrated using the fourth order Runge-Kutta

method from $\eta = 0$ to $\eta = \eta_{\max}$ over successive

step lengths 0.001. Here, η_{\max} is the value of η at ∞ and chosen large enough so that the solution

shows little further change for η larger than η_{\max} .

The accuracy of the assumed values for $M(0)$, $N(0)$, $P(0)$ and $Q(0)$ are then checked by comparing

the calculated values of L , g , θ and ϕ at $\eta = \eta_{\max}$ with their given value at $\eta = \eta_{\max}$. If a difference exists, another set of initial values for $M(0)$, $N(0)$, $P(0)$ and $Q(0)$ are assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at $\eta = \eta_{\max}$ is within the specified degree of accuracy 10^{-4} .

In order to assess the accuracy of the numerical method, we have compared our skin friction component and Nusselt number with those of Abdus Sattar [13] in the absence of mass buoyancy parameter, rotational parameter and species concentration equation without porous medium as shown in the table-1 and we found that excellent agreement among the results exists. Therefore, the developed code can be used with great confidence to study the problem considered in this paper.

Results and discussions

The set of equations (16) – (19) with boundary conditions (20) are solved by employing shooting method that uses Runge – Kutta fourth order method and Newton Raphson method. The results are reported for various values of the physical parameters graphically and presented in figures 2 – 13. The variation of rotational parameter on primary and secondary velocity, temperature and concentration profiles are shown in figures 2 – 5. From fig. 2 we say that increasing rotational parameter retards the velocity distribution, hence primary velocity boundary layer thickness decreased with increase in rotational parameter. We noticed from Figure 3 that secondary velocity increased initially upto reach certain point and then reduced until reach zero at free stream region with increase in rotational parameter. From figs. 4 and 5 we observe that thermal and solutal boundary layer thickness enhanced considerably with increase in rotational parameter.

Figures 6 – 9 depicts the variation of primary and secondary velocity, temperature and concentration distributions respectively for different values of thermophoretic parameter. As increase in thermophoretic parameter primary velocity decreases significantly and hence reduces hydrodynamic boundary layer thickness (fig. 6). From fig. 7 we notice that secondary velocity profile increase with increase in thermophoretic parameter. Increase in thermophoretic parameter leads to enhance temperature distributions conversely concentration results more pronounced and are decreased with increase in thermophoretic parameter as given in figs. 8 and 9.

The effect of suction parameter on primary and secondary velocity, temperature and concentration profiles are shown in figures 10 – 13. It can be seen from figure 10 that increase in suction parameter (fw) enhance primary velocity profile for both positive and negative values of fw . Hence it stabilizes the enhancement of boundary layer thickness. We observe from fig. 11 that secondary velocity profile decreased with increase in suction parameter (fw) for both $fw > 0$ and $fw < 0$. For $\eta=1.8$, it is found that the velocity profile is to decrease and reaches a minimum value near the surface and then gradually increases to zero at free stream region. In figures 12 and 13, the temperature and concentration field for different values of suction parameter are shown. It is seen from these figures that temperature and concentration fields are increased uniformly with increase in suction parameter (fw) for both $fw > 0$ and $fw < 0$. Table-1 represents the variation of thermophoresis parameter and suction/injection parameter on local skin friction components, Nusselt number and Sherwood number. Increasing thermophoresis parameter tends to decrease Nusselt number while enhance skin friction component τ_x , τ_z and Sherwood number. As increase in suction/injection parameter, local skin friction components τ_x and τ_z are increased while Nusselt number and Sherwood number reduces significantly.

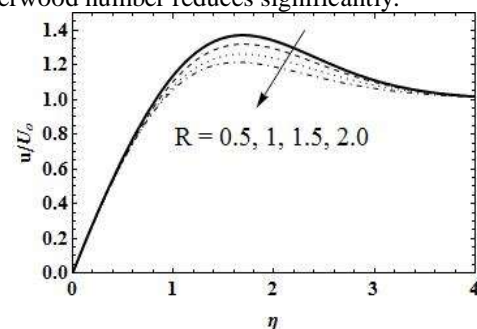


Fig 2: Primary Velocity profile for different values of R

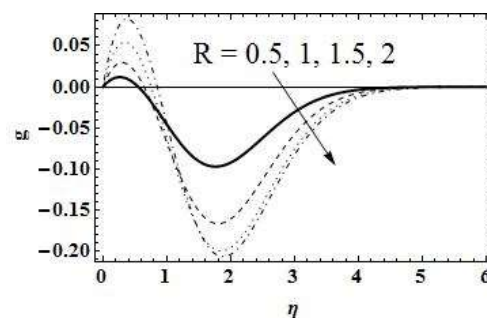


Fig 3: Secondary Velocity profile for different values of R

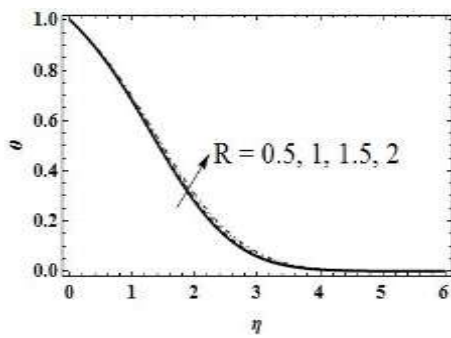


Fig 4: Temperature profile for different values of R

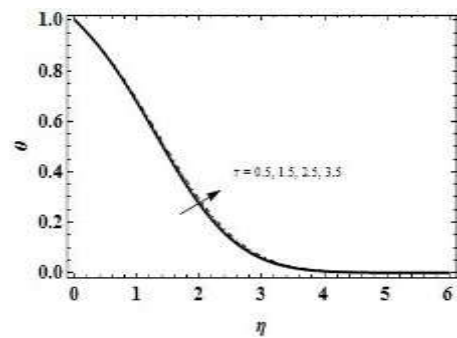


Fig 8: Temperature profile for different values of τ

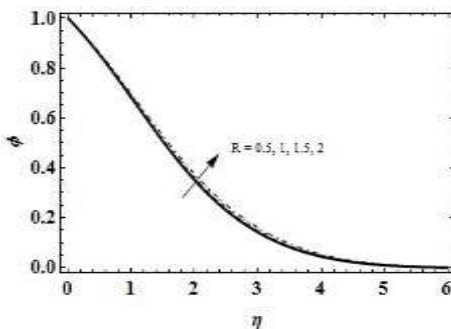


Fig 5: Concentration profile for different values of R

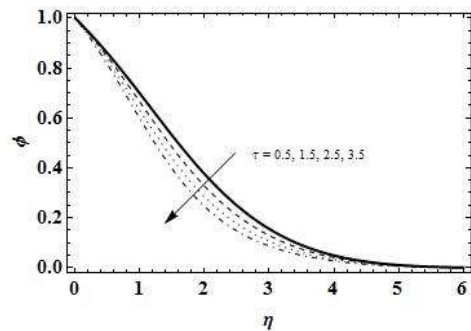


Fig 9: Concentration profile for different values of τ

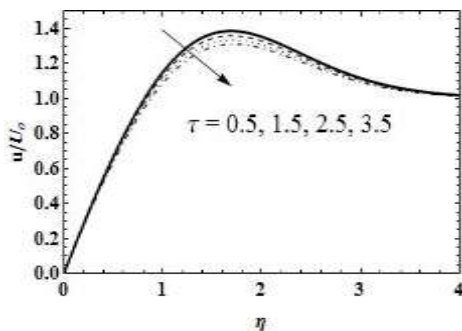


Fig 6: Primary Velocity profile for different values of τ

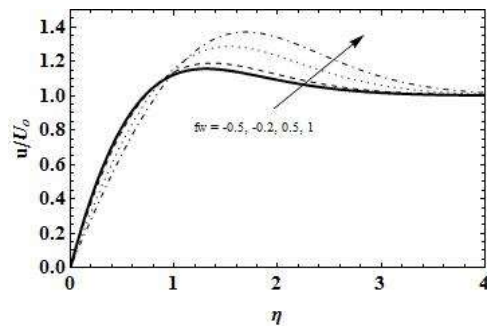


Fig 10: Primary Velocity profile for different values of f_w

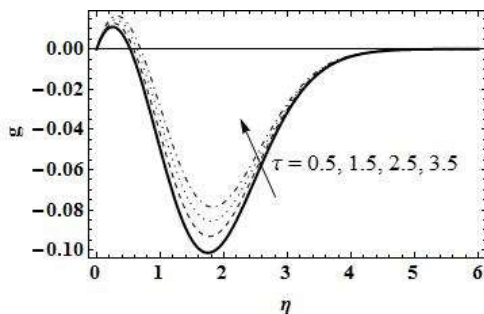


Fig 7: Secondary Velocity profile for different values of τ

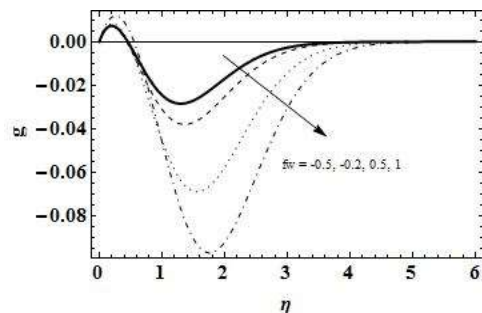


Fig 11: Secondary Velocity profile for different values of f_w

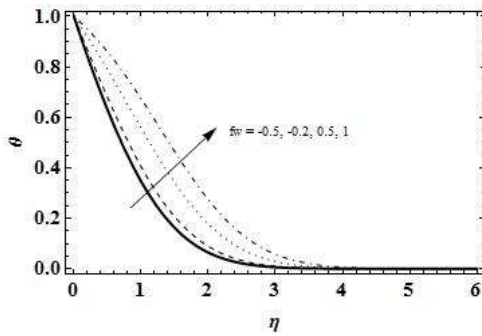


Fig 12: Temperature profile for different values of fw

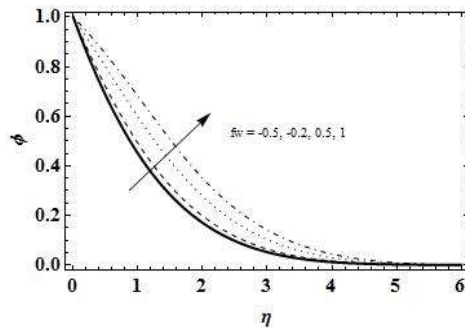


Fig 13: Concentration profile for different values of fw

Table-1: Comparison values of skin friction component and Nusselt number with the results of Abdus Sattar [13] in the absence of concentration equation, $R=0$, $\tau=0$, and $g_c=0$.

Parameters		Abdus Sattar [13]		Present Results	
Fw	g _s	τ_x	Nu	τ_x	Nu
2	0.5	3.5905	1.2749	3.59045	1.27488
2	1.0	3.9894	1.0254	3.98952	1.02539
2	5.0	7.1808	0.2572	7.18076	0.25732

Table-1: Values of the local skin friction components τ_x, τ_z , Nusselt number and Sherwood number for different values of thermophoresis parameter and suction/injection parameter for $G=1, N=1, R=0.5, Pr=0.71$, and $Sc=0.32$.

Fw	τ	τ_x	τ_z	Nu	Sh
1	0.5	-1.48147	0.09163	0.236136	0.25532
1	1.5	-1.4645	0.09759	0.2327	0.27463
1	2.5	-1.4479	0.10324	0.22942	0.29761
1	3.5	-1.43173	0.10858	0.22631	0.32400
-0.5	1	-2.33105	0.08159	0.81285	0.72025
-0.2	1	-2.15146	0.07994	0.66931	0.61052
0.5	1	-1.74745	0.08331	0.38618	0.38922
1.0	1	-1.47295	0.09464	0.2344	0.26451

Conclusions

1. Increasing rotational parameter tends to decrease primary and secondary velocities while reduce the temperature and concentration fields.
2. With increase in thermophoretic parameter cause to reduce primary velocity and concentration field variable while enhance the secondary velocity and temperature profile.
3. Increase in suction parameter enhance primary velocity, temperature and concentration profile while decelerate the secondary velocity.

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